3.11.1. Duality Problems

A. Consider the formal language **{ }** which has **no connectives**, only sentence letters. What is the **dual** of language **{ }**?

B. Construct formal sentences (possibly with truth tables) to do each of the following.

1. Show that the language {↔} has a sentence matching the truth table for "⊤".

2. On the basis of language duality and your answer to (1), show that the language {⊕} has a sentence matching the truth table of "⊥".

3. Show that the language {↔, ⊥} has a sentence matching the truth table for "~P".

4. On the basis of Tilde Insertion duality and your answer to (3), show that the language {↔, ⊥} has a sentence matching the truth table for "(P ⊕ Q)".

5. On the basis of your answers to (2) and (3), show that the language {↔, ⊕} has a sentence matching the truth table for "~P".

6. On the basis of language duality and your answer to (3), show that the language {⊕, ⊤} has a sentence matching the truth table for "~P".

7. On the basis of Tilde Insertion duality and your answer to (6), show that the language {⊕, ⊤} has a sentence matching the truth table for "(P ↔ Q)".

8. On the basis of language duality and your answer to (4), show that the language {⊕, ⊤} has a sentence matching the truth table for "(P ↔ Q)".

C. State, for each of the following sets of sentences, if that set is **closed under semantic duality**.

1. The set of consistent sentences.
2. The set of sentences which are neither a tautology nor a contradiction.
3. The set of Chapter Two sentences.
4. The set of sentences of the language {∧, ∨}.
5. The set of sentences of the language {∧, ~}.
6. The set of negations in the Chapter Two language.
7. The set of negations in the Chapter Three language.